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14. ABSTRACT
The grant was to develop new models and Bayesian statistical inference for Cultural Consensus Theory (CCT). CCT is an approach to pooling response data from informants (experts) to estimate their consensus knowledge, unknown apriori to the researcher. During the period of the grant, the PI made progress in all five areas of the proposed research, namely (1) Constructing a catalog of CCT models for different testing formats; (2) Developing and implementing Bayesian computational inference for the models; (3) Determining the minimal number of informants needed to achieve confidence in the pooled information; (4) Developing an approach to aggregating expert views of the ties in a digraph that imposes prior constraints on the consensus digraph; (5) Developing CCT models that detect cultural variation among the informants. Progress on the grant was evidenced by seven new publications on CCT, fifteen invited talks on CCT by the PI, two new grants on CCT, one from the Army Research Office (ARO) and the other from the Intelligence Advanced Research Projects Activity (IARPA), a connection with RAND at Santa Monica involving CCT, and the a

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FA9550-09-1-0510

The original grant proposal was to advance Cultural Consensus Theory (CCT) as an information pooling (aggregation, fusion) tool, and to examine possible areas where it might be applicable to military science. The proposal was to conduct research in five main project areas all involving the development of CCT. The main body of this report will describe these project areas and the work that has been done in each of them. However, first some background information about CCT will be provided.

The PI, Batchelder, and A. Kimball Romney invented CCT in the 1980s (Batchelder & Romney, 1986, 1988, 1989; Romney, Batchelder, and Weller, 1987; Romney, Weller, & Batchelder, 1986), and its further development has been supported between 1986 and 2000 by several grants from the National Science Foundation to Romney and Batchelder, Co-PIs.

Data for CCT consists of the responses of informants (experts, eyewitnesses, automated sources, or members of a group) to a set of questions about some domain of their shared knowledge. Validated cognitive models are used to estimate the consensus knowledge of the informant group as well as the competence (degree of cultural knowledge) and response bias characteristics of each informant.

As a consequence of this earlier work, one of the CCT models for dichotomous True/False (Yes/No) response data has been in wide use to determine consensus cultural views in cognitive and medical anthropology (see Weller, 2007). The model, known as the General Condorcet Model (GCM), derives from assumptions in signal detection theory and psychometric test theory, except that unlike models in these two areas, the GCM does not assume that the researcher has access to the correct (consensus) answers (signal or noise in the case of signal detection, and correct or error in the case of test theory). Instead the GCM postulates a consensus answer key $\mathbf{Z} = < Z_k >$ as a vector of parameters in the model, where $Z_k = 1$ if the consensus answer to question k is 'True' and $Z_k = 0$ if 'False'. In addition the model postulates that each informant *i* has a competence (ability) parameter, D_i which specifies the probability that he or she detects the correct consensus answer to a question, and in addition each informant has a guessing bias parameter, g_i, which governs the probability of a 'True' response when the correct answer is not detected. The details of the model are presented in Batchelder and Romney (1988) and Karabatsos and Batchelder (2003), and it is reviewed and expanded in Batchelder and Anders (2012).

The main available software package used to estimate the GCM was developed in the early 1990s and is found in ANTHROPAC (Borgatti, 1996) and UCINET (Borgatti, Everett, & Freeman, 2002). Unfortunately, this software only provides point estimates of the competence parameters and the answer key parameters of a simplified version of the GCM that sets the guessing bias to a constant 0.50 for all informants. This latter assumption is especially of concern because in all applications of signal detection models it is well know that different respondents have different response bias characteristics. In Batchelder and Romney (1988), the full GCM was developed that allowed heterogeneity in informants, guessing bias as well as heterogeneity in item difficulty (cultural salience); however, it was much later until Bayesian inference software was developed in S by Karabatsos and Batchelder (2003) to estimate the full GCM. Unfortunately, Anthropologists continue to use the earlier software in ANTHROPAC and UCINET, and they have not acquired the

programming skills or acceptance of Bayesian inference theory needed to adopt that software.

The current grant was primarily to develop new CCT models for different questionnaire formats as well as to establish state-of-the-art Bayesian inference for all the models including the GCM. The grant proposal described five main project areas as follows: (1) Constructing a catalog of CCT models for different testing formats; (2) Developing and implementing Bayesian computational inference for the models; (3) Determining for each testing format the minimal number of informants needed to achieve confidence in the pooled information; (4) Developing an approach to aggregating expert views of ties in a digraph that imposes prior constraints on the consensus digraph; (5) Developing CCT models that detect cultural variation and/or prevarication among informants.

In the two plus years of the grant, major progress has been made in project areas (1), (2), and (4), and some progress has also been made in project areas (3) and (5). The next few sections of this final report will describe the progress in these five project areas in detail. In addition, work supported by this grant has led to two additional grants: (1) A three year grant to Batchelder (PI) starting in the fall of 2010 from the Army Research Office (ARO) to develop a Bayesian inference tool kit for CCT models that is both user friendly and freely available to researchers, and (2) A multi-site grant from the Intelligence Advanced Research Projects Activity (IARPA) to augment the probabilistic prediction accuracy of government forecasters. That grant is funneled through Applied Research Associates (ARA) and includes faculty researchers from several universities. Batchelder is a researcher on this grant, and Mark Steyvers is the UCI PI.

Project Area 1. Constructing a Catalogue of CCT Models.

As mentioned above, Prior to the grant from the AFOSR the main CCT model in use was the GCM. That model enabled researchers to ask a series of True/False questions to the members (informants) of a group who share cultural knowledge, and to use the data to infer the consensus answers to the questions as well as the relative cultural competence (level of cultural knowledge) of the informants. In addition, that model was modified to handle multiple-choice items, and an informal CCT approach was designed by Romney, Batchelder, and Weller (1987) to handle rank order response data.

In the period supported by the AFOSR grant new CCT models have been developed for a number of new response modes: (1) dichotomous response data based on fuzzy (continuous) rather than crisp truth values by Batchelder and Anders (2012) (2) reporting the ties in a social network represented as a digraph by Batchelder, (2009); (3) reporting the sign of ties in a signed graph by Agrawal and Batchelder (2012); (4) continuous responses in the unit interval (as in probability estimation) by Batchelder, Strashny, and Romney (2010); (5) matching a response set to stem items (e.g. flags to countries) by Batchelder and Steyvers (in preparation, reported in several invited presentations) and by Zeigenfuse and Batchelder (in preparation, reported in several presentations); (6) responses to items in an ordered categorical (Likert) scale (Anders and Batchelder (2012, in preparation); (7) judges rank ordering competitors by Anders and Batchelder, (2012, in preparation).

The ‘in preparation’ papers all have first drafts, but additional work is needed before they can be submitted to a journal for publication. These projects have gone a long way to increasing the number of models for different response types in CCT. Some of these new models have been published with

new data sets, and others are developed and can handle recovery of parameters in simulated data but we are still seeking suitable real data to try them out before submitting them to a journal.

Project Area 2. Developing and Implementing Bayesian Computational Inference for the Models.

Perhaps the area of the most development during the years of the grant has been the development of Bayesian inference software for various CCT models. The basic work was well underway during the earlier years of the grant from the AFOSR, and it picked up greatly with the addition of the three year grant from the Army Research Office (ARO) that enabled the PI to hire a post doctoral fellow with considerable expertise in Bayesian computational statistics, and to support several graduate students (see list of laboratory personnel dealing with CCT in a later section).

As mentioned, Bayesian inference for the GCM is in Karabatsos and Batchelder (2003). That paper developed a Markov Chain Monte Carlo (MCMC) sampler for the GCM in the S language (a forerunner of R). It was a hybrid Metropolis-Hastings and Gibbs sampler, and code, but no interface, was offered to potential users. Unfortunately only a few researchers have used the software, and instead they have continued to use CCT software developed in the late 1980s and provided in ANTHROPAC (Borgatti, 1996) and UCINET (Borgatti, Everett, & Freeman, 2002). That software was developed using results in Batchelder and Romney (1988). The key to the software is to obtain a point estimate of each informant's competency using a methods-of-moments approach, and then use these point estimates along with Bayes theorem to obtain a posteriori distribution over the possible answer keys. There are two salient disadvantages in using that software. First, as mentioned, it only applies to a special case of the GCM for T/F items, where there is no informant heterogeneity in guessing bias and there is no heterogeneity in item difficulty. Second, with only point estimates of the competencies of the informants, there is no way to assess the confidence in these point estimates. The Karabatsos and Batchelder (2003) solved these problems; however, as stated earlier, the social science community that uses CCT has not adopted that approach. One of the main thrusts of the grant was to correct this situation.

All of the CCT papers during the duration of the grant have used Bayesian inference via MCMC samplers. Some of the models were sufficiently complex that we had to develop our own MCMC samplers, e.g. Batchelder, Strashny, and Romney (2010), and Agrawal and Batchelder (2012). However, the major breakthrough during the grant was to show that standard, free access software packages could handle Bayesian inference for the most general version of the GCM. In particular, we developed software with JAGS that uses R in Batchelder and Anders (2012), and more recently we have designed a software package for the GCM that uses WinBUGS (Lund, Thomas, Best, & Spiegelhalter, 2000). That package is described in Oracevz, Vandekerchove, and Batchelder (2012). Both papers are under review. The latter paper designs the package BCCT (Bayesian Cultural Consensus Toolbox) that can analyze eight different versions of the GCM depending on whether or not competence, guessing bias, and item difficulty are heterogeneous. It is designed to be very user friendly, and the paper is designed to allow a reader to download and use the package.

Project Area 3. Determining for each Testing Format the Minimal Number of Informants Needed to Achieve Confidence in the Pooled Information

In Batchelder and Romney (1988), power tables were provided for the GCM (the CCT model for T/F items) that gave the minimum number of informants needed to achieve a satisfactory recovery of the consensus answers to the T/F items as a function of the average competence and pre-specified levels of confidence. One surprising aspect of this table is that far fewer informants are needed to achieve a particular level of confidence in the recovery of the consensus answers based on a majority rule with confidence intervals from the binomial theorem. This is because the GCM endogenously weights the informants by their competence, and gives more weight to the more competent informants. As shown in Batchelder and Romney (1988) this leads to log-odds aggregation rather than simple majority rule.

The goal of this project area is to develop similar tables for other CCT models. As mentioned in Project Area #1, several new CCT models have been developed during the grant. We have conducted numerous simulations using Bayesian software on simulated data for these models. These simulations have varied the number of informants and their average competence toward the aims of this Project Area. While we are not yet able to present detailed tables for the new models in the form of the table for the GCM in Batchelder and Romney (1988), it is clear from this work that for response models more complex than dichotomous T/F items, even fewer informants are necessary to achieve accurate reconstruction of the consensus answers.

In fact, if the assumption of a single consensus truth is valid for a given set of response data, once the number of informants exceeds ten or so, one can operate with a CCT model at about the same level of precision as would be had in case one had the consensus answers apriori. For this reason it has become of paramount impotents to develop tests of the single culture assumption. Such a test was developed as a Bayesian post predictive test in Batchelder and Anders (2012). The test is based on a property of the GCM proved in Batchelder and Anders (2012) that the correlation between two informants, *i* and *j*, over their responses to the items, ρ_{ij}, satisfies the formula $\rho_{ij} = \rho_{iZ}\rho_{jZ}$, where the right hand side is the product of the correlation of each informant with the unknown consensus answers. Such a fact is a version of Spearman's famous tetrad law, $\rho_{ij}\rho_{kl} = \rho_{il}\rho_{kj}$, concerning correlations between tests across test takers that is behind his two factor theory of intelligence. In essence this property for CCT models says that the sole basis of any correlation between informants is due entirely to the fact that they are hypothesized to share a common consensus answer key. This result suggests that if the informant-by-informant correlations are subjected to a factor analysis, that the signal in the data will be represented in the first factor as shown above. The Bayesian post predictive check of the GCM model compares the ratio of the first to second eigenvalues of the factor analysis for the real data against the same ratio obtained by simulating thousands of simulated data sets obtained during the MCMC sampler. The property of a single consensus answer key stands or falls with rather or not the ratio in the real data falls midway in the distribution of ratios from the simulated data sets.

Project Area 4. Developing an approach to aggregating expert views of ties in a digraph that imposes prior constraints on the consensus digraph.

Batchelder, Kumbasar, and Boyd (1997) and Batchelder (2009) provide CCT models for pooling responses from experts about the arcs in a digraph. In these models, there are no prior constraints on the nature of the responses of the experts such as symmetry, transitivity, or structural balance. Imposing constraints on the consensus graph, without at the same time imposing them on the

experts' responses leads to the necessity of developing a special MCMC sampler to do Bayesian inference. This is because available software packages like JAGS and WinBUGS cannot impose structural constraints in graphs in their samplers.

In the period under review we have worked with signed graphs (graphs that have symmetric ties that are either positive or negative, such as friend or enemy). Agrawal and Batchelder (2012) present a model for the case where each expert provides a complete signed graph (there is a signed tie between every pair of distinct nodes), and the consensus graph is required to satisfy Cartwright and Harary's definition of balance in a social network. This sense of balance is that the graph nodes can be partitioned into two sets, one of which may be empty, where ties between nodes in the same set are positive and ties between nodes in different sets are negative. If the graph has N nodes, it is easy to show that there are $2^{N(N-1)/2}$ distinct complete signed graphs, and only 2^{N-1} of them satisfy this sense of balance. The model new model was used to analyze real and simulated data, and the MCMC sampler that was developed is able to recover the expert and item parameters as well as the consensus balanced graph. This work has been presented in three conference papers, two at the 2012 Annual Sunbelt Social Network conference and one at the 2012 Annual Social Computing, Behavioral Cultural Modeling, and Prediction conference.

The special concern in imposing constraints on a consensus graph structure is that there must be a way to search the parameter space of graphic structures that satisfy the constraint. This can lead to complex combinatorial issues. For example, the work in Agrawal and Batchelder (2012) is being generalized to Davis' sense of balance, where the partition can have more than two cells, where as before positive ties are within cells and negative ties between cells. In this case the so-called Bell number gives the number of partitions on a finite set, and this number can only be expressed recursively. Similar combinatorial complexity issues occur with other constraints such as transitivity or a partial order.

It is supposed that CCT models for pooling graphs under constraints may be useful in detecting intelligence applications, for example detecting covert networks. In such a case, the intelligence experts will use their knowledge to fill in the arcs in a covert network defined by 'giving orders to,' 'meeting secretly with', or 'sharing information with.' Then one would impose certain organizational structures on the consensus graph based on intelligence reports about the likely structure of a particular covert network, for example a central node representing the coordinator with arcs to several others to the members, but no other arcs between the members to preserve secrecy. The value of the approach is that even though the experts' graphic responses do not satisfy the supposed constraints, perhaps due to incomplete knowledge, the resulting consensus graph will represent the most likely covert structures. It is planned to seek further funding to carry out this program. It will involve complex issues in combinatorial complexity and special MCMC samplers.

Project Area 5. Developing CCT models that detect cultural variation and/or prevarication among informants.

Most of the CCT models are designed for the situation where all the informants share the same consensus answers to the questions. This crucial one-culture property of the GCM for dichotomous T/F questions is essential for the Bayesian inference for the model provided in Karabatsos and Batchelder (2003), and it lead to an important Bayesian post predictive check developed in

Batchelder and Anders (2012) discussed earlier in project area 3.

The most natural way to relax the one-culture assumption and allow for cultural variation is to augment the model to allow two or more consensus answer keys. The formalization of this idea as a finite mixture model was presented in Batchelder and Romney (1989); however, no complete statistical inference was provided. Their approach to augmenting the GCM was to add two or more answer keys along with informant membership parameters that indicate which answer key belongs to which informant.

In the period under review, the GCM model developed earlier has been reworked so that Bayesian inference for the model allowing up to three answer keys can be conducted with JAGs, the same Bayesian software package used in Batchelder and Anders (2012) to estimate the GCM with a single consensus answer key. So far a draft of a paper has been written and a data set involving several answer keys is being analyzed. We expect that this paper will be completed and submitted for publication by June 2012.

Crucial in this work is to develop a diagnostic for the number of answer keys that are needed to best fit the data. Toward this goal, a new theorem for the multi-culture GCM has been derived. The theorem concerns the correlations between pairs of informants over the items, ρ_{ij} introduced earlier in progress area 3. The result is for all pairs of informants *i* and *j*, $\rho_{ij} = \rho_{iZ^{(i)}}\rho_{Z^{(i)}Z^{(j)}}\rho_{jZ^{(j)}}$. In this formula the first and last term refer to the correlation between one of the informants' responses to the items and his or her own consensus answer key, and the middle term is the correlation between the two answer keys. In fact this formula reduces to the one described in project area 3 if the two informants share the same consensus answer key. We have constructed a test of multi-cultures based on this formula. The idea is that when the informant-by-informant correlation matrix is factored (see project area 3), the first factor will underestimate correlations between informants with the same consensus answer key and overestimate the correlation between informants with different answer keys. Based on this, it is possible to cluster the informants based on the second residual matrix after the first factor is pulled out of the original correlation matrix. We are working this idea into a new Bayesian post predictive check for the multi-culture version of the GCM.

While the expansion of the model to handle multiple answer keys enables us to detect cultural variation when there is more than one informant in a cultural group, we have made less progress in detecting prevarication in informants as described in the second part of the goals of project area 5. Currently we are working with a psychometrician to adapt an approach used in item response theory (IRT) called differential item functioning (DIF) that is designed to find respondents who have different response probabilities than the large majority of the other respondents. Our goal is to discover a 'lie detection statistic' that can be used to screen the informant pool. We are still working on this problem at this time.

In short, there has been a great deal of progress in the five project areas of the grant, and this has led to a number of papers, invited conference and workshop presentations, several additions to the PIs lab, and two new grants concerning CCT.

Personal During the AFOSR Grant

Because of the support of the grant from the AFOSR, I have added several graduate students and post doctoral researchers to my laboratory who I advise and are working on CCT projects. I list them and any change in their status during the period of the grant.

1. Royce Anders- Passed his PhD Candidacy Exam in Department of Cognitive Sciences, currently a fourth year student.
2. Gregory Alexander- Started as an Undergraduate Researcher on the grant and is now a first year graduate student in the PhD Program of the Department of Cognitive Sciences.
3. Kalin Agrawal- Third year graduate student in the Mathematical Behavioral Sciences PhD Program.
4. Giorgio Gosti- Passed his PhD Candidacy Exam in the Mathematical Behavioral Sciences PhD Program.
5. Dr. Zita Oravecz, Post Doctoral Fellow who's PhD was from University of Leuven, Belgium, supported by the grant from Army Research Office described earlier.
6. Dr. Stephen France, Assistant Professor Market Research University of Wisconsin Milwaukee. Sabbatical to learn about CCT and apply it to market researach

Publications During the AFOSR Grant (* indicates ones directly concerned with CCT)

*Batchelder, W. H. (2009). Cognitive Pscyometrics: Using Multinomial Processing Tree Models as Measurement Tools. In S. E. Embretson, *Measuring Psychological Constructs: Advances in Model Based Measurement* (pp. 71-93). Washington D.C.: American Psychological Association Books.

*Batchelder, W. H. (2009). Cultural Consensus Theory: Aggregating Expert Judgments about Ties in a Social Network. In H. Liu, J. Salemo, & M. J. Young, *Social Computing, Behavioral Modeling and Prediction* (pp. 24-32). New York: Springer.

Batchelder, W. H., Hu, X., & Smith, J. B. (2009). Multinomial Processing Tree Models for Discrete Choice. Special Issue on New Developments in Multinomial Process Tree Modeling. *Zeitschrift für Psychologie , 217*, 149-158.

Purdy, B., & Batchelder, W. H. (2009). A Context-free Language for Binary Multinomial Processing Tree Models. *Journal of Mathematical Psychology , 53*, 547-561.

Batchelder, W. H. (2010). Mathematical Psychology. In L. Nadel, *Wiley Interdisciplinary Reviews: Cognitive Science* (pp. 759-765). New York: Wiley.

*Batchelder, W.H., Strashny, A., and Romney, A.K. (2010) Cultural Consensus Theory: Aggregating Continuous Responses in a Finite Interval. In S.-K. Chai, J.J. Salerno, and P.L. Mabry (Eds.). *Social Computing, Behavioral Modeling, and Prediction 2010* (pp. 98-107).New York: Springer, 2010, pp. 98-107.

Smith, J.B. and Batchelder, W.H. (2010). Beta-MPT: Multinomial processing tree models for addressing individual differences. *Journal of Mathematical Psychology, 54*, 167-183.

Wu, H., Myung, J.I., and Batchelder, W.H. (2010). On the minimum description length complexity of multinomial processing tree models. *Journal of Mathematical Psychology*, *54*, 291-303.

Wu, H., Myung, J.I., and Batchelder, W.H. (2010). Minimum description length model selection of multinomial processing tree models. *Psychonomic Bulletin & Review*, *17*, 275-286.

Schmittmann, V.D., Dolan, C.V., Raijmakers, M.E.J., and Batchelder, W.H. (2010). Parameter identification in multinomial processing tree models. *Behavior Research Methods*, *42*, 836-846.

Batchelder, W.H. (2010). Mathematical Psychology. In L. Nadel (Ed.).*Wiley Interdisciplinary Reviews: Cognitive Sciences* (pp. 759-765). New York: Wiley.

*Gosti, G., and Batchelder, W.H. (2011). Naming on a directed graph. In J.J. Salerno, J.Y. Shanchieh, D.S. Nau, and S-K Chai (Eds.). *Social Computing, Behavioral-Cultural Modeling and Prediction LNCS 6589* (pp. 358-365). New York: Springer Verlag.

*Agrawal, K., and Batchelder, W.H. (2012). Cultural consensus theory: Aggregating signed graphs under a balance constraint. In S.J. Yang, A. M. Greenberg, and M. Endsley (Eds.). *Social Computing, Behavioral-Cultural Modeling and Prediction*, *LNCS 7227* (pp.53-60). New York: Springer Verlag.

Batchelder, W.H., Hu, X., and Riefer, D.M. Multinomial Modeling. In H. Pashler (Ed.). The Encyclopedia of the Mind. Sage Publications, in press.

Invited Conference and Workshop Presentations on CCT During the AFOSR Grant

Batchelder, W.H. Invited paper on Cultural Consensus Theory read at AFOSR Program Review of Mathematical Modeling of Cognitive and Decision Processes. Arlington Vg., January 2009.

Batchelder, W.H. Cultural Consensus Theory: Aggregating expert judgments about ties in a Social Network. Invited paper Read at 2ND Annual Workshop on Social Computing, Behavioral Modeling, and Prediction. Phoenix, Az. April 2009.

Batchelder, W.H. Cultural Consensus Theory: New models for continuous response and matching tests. AFOSR Joint Review Cognition and Decision Making Program and Human-System Interface Program. Arlington, Vg., January 2010.

Batchelder, W.H., Strashny, A., and Romney, A.K. Cultural Consensus Theory: A model for a continuous responses in a finite interval. Invited Paper read at the Annual Conference on Social Computing, Behavioral Modeling, and Prediction. NIH Campus, Bethesda, Md. April 2010.

Batchelder, W.H. Cultural Consensus Theory. Invited paper read to RAND Corporation Centers, Santa Monica, Ca. July 2010.

Batchelder, W.H. Cultural Consensus Theory. Invited paper read in Symposium on Wisdom of the Crowd, Annual Meeting of the Society for Mathematical Psychology, Portland, Or. August 2010.

Anders, R. (Presenter), and Batchelder, W.H. Rank-Aggregation and consensus in Ballroom Competition. Paper read at Annual Meeting of the Society for Mathematical Psychology, Portland, Or. August 2010.

Zeigenfuse, M. (Presenter), Batchelder, W.H., and Steyvers, M. A three parameter Item Response Model of Matching. Paper read at Annual Meeting of the Society for Mathematical Psychology, Portland, Or. August 2010.

Batchelder, W.H. Statistical Development and Applications of Cultural Consensus Theory. Paper presented at AFOSR Joint Review Cognition and Decision Making Program. Dayton, Ohio, January 2011.

Batchelder, W.H. Observations about Cultural Consensus Theory. Paper presented at Workshop on Dynamic Models of Cultural Diversity. Arizona State University, Tempe, Az. February, 2011

Gosti, G. (presenter), and Batchelder, W.H. Naming on a directed graph. Poster presented at the Conference on Social Computing, Behavioral-Cultural Modeling, and Prediction. University of Maryland, College Park, Md. March 2011.

Batchelder, W. H., and Anders, R. Cultural Consensus Theory: Comparing different concepts of cultural truth. Paper presented at Annual Meeting of the Society for Mathematical Psychology. Tufts University, July 2011.

Gosti, G. (presenter), and Batchelder, W.H. The Naming Game on a Directed Graph. Paper presented at Annual Meeting of the Society for Mathematical Psychology. Tufts University, July 2011.

Agrawal, K. (presenter), and Batchelder, W.H. Cultural Consensus Theory: Estimating Consensus Graphs under constraints. Paper presented at Annual Meeting of the Society for Mathematical Psychology. Tufts University, July 2011.

Batchelder, W.H. Cultural Consensus Theory: Detecting Experts and Their Shared Knowledge. Invited Paper presented at DIMACS Workshop on the Science of Expert Opinion. Rutgers University, October, 2011.

Papers Under Review on CCT

Batchelder, W.H., and Anders, R. (2012). Cultural consensus theory: Comparing different concepts of cultural truth. *Journal of Mathematical Psychology*_(under review).

Oravecz, Z., Vandekerckhove, and Batchelder, W. H. (2012). Bayesian cultural consensus theory. *Field Methods* (under review)

Additional Research Opportunities Made Possible by the Grant.

There were several research connections between our lab and others that were facilitated by the grant. They are covered in the next few paragraphs.

Researchers at RAND in Santa Monica expressed interest in the CCT project. They invited the PI, Batchelder, to give an hour talk on CCT that was broadcasted to all the divisions of RAND, Batchelder, W.H. Cultural Consensus Theory. Invited paper read to RAND Corporation Centers, Santa Monica, Ca. July 2010. In addition, RAND hired Kalin Agrawal, an advanced PhD student in the PIs lab for a summer internship at RAND.

The Army Research Office representatives heard a talk by the PI on CCT, and after some correspondence they funded a three-year grant for over $300,000 that is in its second year. This grant allowed the hiring of a Post Doctoral fellow, Zita Oravecz, who has considerable skill with software and Bayesian statistics. This has led to the software package, BCCT, described in Oravecz, Vandekerckhove, and Batchelder (2012).

An Institute for Cultural Studies at Arizona State University funded a Workshop on Dynamic Models of Cultural Diversity. The PI was a keynote presenter to acknowledge the 25th anniversary of the development of CCT. The efforts of the researchers at this workshop have led to funding opportunities for the group with NSF. The invited presentation was: Batchelder, W.H. Observations about Cultural Consensus Theory. Paper presented at Workshop on Dynamic Models of Cultural Diversity. Arizona State University, Tempe, Az. February, 2011.

A multi-site grant for several million dollars was received from the Intelligence Advanced Research Projects Activity (IARPA) to augment the probabilistic prediction accuracy of government forecasters. That grant is funneled through Applied Research Associates (ARA) and includes faculty researchers from several universities. Batchelder is a researcher on this grant, and Mark Steyvers is the UCI PI.

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